

## Fluid Mechanics

**Problem C****PRESSURE AS A FUNCTION OF DEPTH****PROBLEM**

In 1969, a whale dove and remained underwater for nearly 2 h. Evidence indicates that the whale reached a depth of 3.00 km, where the whale sustained a pressure of  $3.03 \times 10^7$  Pa. Estimate the density of sea water.

**SOLUTION**

**Given:**

$$h = 3.00 \text{ km} = 3.00 \times 10^3 \text{ m}$$

$$P = 3.03 \times 10^7 \text{ Pa}$$

$$P_o = 1.01 \times 10^5 \text{ Pa}$$

$$g = 9.81 \text{ m/s}^2$$

**Unknown:**  $\rho = ?$

Use the equation for fluid pressure as a function of depth, and rearrange it to solve for density.

$$P = P_o + \rho gh$$

$$\rho = \frac{P - P_o}{gh}$$

$$\rho = \frac{(3.03 \times 10^7 \text{ Pa}) - (1.01 \times 10^5 \text{ Pa})}{(9.81 \text{ m/s}^2)(3.00 \times 10^3 \text{ m})}$$

$$\rho = \frac{3.02 \times 10^7 \text{ Pa}}{(9.81 \text{ m/s}^2)(3.00 \times 10^3 \text{ m})}$$

$$\rho = \boxed{1.03 \times 10^3 \text{ kg/m}^3}$$

**ADDITIONAL PRACTICE**

- In 1994, a 16.8 m tall oil-filled barometer was constructed in Belgium. Suppose the barometer column was 80.0 percent filled with oil. What was the density of the oil if the pressure at the bottom of the column was  $2.22 \times 10^5$  Pa and the air pressure at the top of the oil column was  $1.01 \times 10^5$  Pa?
- One of the lowest atmospheric pressures ever measured at sea level was  $8.88 \times 10^4$  Pa, which existed in hurricane Gilbert in 1988. This same pressure can be found at a height 950 m above sea level. Use this information to estimate the density of air.
- In 1993, Francisco Ferreras of Cuba held his breath and took a dive that lasted more than 2 min. The maximum pressure Ferreras experienced was 13.6 times greater than atmospheric pressure. To what depth did Ferreras dive? The density of sea water is  $1.025 \times 10^3 \text{ kg/m}^3$ .
- A penguin can endure pressures as great as  $4.90 \times 10^6$  Pa. What is the maximum depth to which a penguin can dive in sea water?

5. In 1942, the British ship *Edinburgh*, which was carrying a load of 460 gold ingots, sank off the coast of Norway. In 1981, all of the gold was recovered from a depth of 245 m by a team of 12 divers. What was the pressure exerted by the ocean's water at that depth?
6. In 1960, a bathyscaph descended 10 916 m below the ocean's surface. What was the pressure exerted on the bathyscaph at that depth?

## Additional Practice C

### Givens

1.  $h = (0.800)(16.8 \text{ m})$   
 $P = 2.22 \times 10^5 \text{ Pa}$   
 $P_0 = 1.01 \times 10^5 \text{ Pa}$   
 $g = 9.81 \text{ m/s}^2$

### Solutions

$$P = P_0 + \rho gh$$
$$\rho = \frac{P - P_0}{gh} = \frac{2.22 \times 10^5 \text{ Pa} - 1.01 \times 10^5 \text{ Pa}}{(9.81 \text{ m/s}^2)(0.800)(16.8 \text{ m})} = \frac{1.21 \times 10^5 \text{ Pa}}{(9.81 \text{ m/s}^2)(0.800)(16.8 \text{ m})}$$
$$\rho = \boxed{918 \text{ kg/m}^3}$$

2.  $h = -950 \text{ m}$   
 $P = 8.88 \times 10^4 \text{ Pa}$   
 $P_0 = 1.01 \times 10^5 \text{ Pa}$   
 $g = 9.81 \text{ m/s}^2$

$$P = P_0 + \rho gh$$
$$\rho = \frac{P - P_0}{gh} = \frac{8.88 \times 10^4 \text{ Pa} - 1.01 \times 10^5 \text{ Pa}}{(9.81 \text{ m/s}^2)(-950 \text{ m})} = \frac{-1.2 \times 10^4 \text{ Pa}}{(9.81 \text{ m/s}^2)(-950 \text{ m})}$$
$$\rho = \boxed{1.3 \text{ kg/m}^3}$$

3.  $P = 13.6P_0$   
 $P_0 = 1.01 \times 10^5 \text{ Pa}$   
 $\rho = 1.025 \times 10^3 \text{ kg/m}^3$   
 $g = 9.81 \text{ m/s}^2$

$$P = P_0 + \rho gh$$
$$h = \frac{13.6P_0 - P_0}{\rho g} = \frac{12.6P_0}{\rho g}$$
$$h = \frac{(12.6)(1.01 \times 10^5 \text{ Pa})}{(1.025 \times 10^3 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} = \boxed{127 \text{ m}}$$

4.  $P = 4.90 \times 10^6 \text{ Pa}$   
 $P_0 = 1.01 \times 10^5 \text{ Pa}$   
 $\rho = 1.025 \times 10^3 \text{ kg/m}^3$   
 $g = 9.81 \text{ m/s}^2$

$$P = P_0 + \rho gh$$
$$h = \frac{P - P_0}{\rho g} = \frac{4.90 \times 10^6 \text{ Pa} - 1.01 \times 10^5 \text{ Pa}}{(1.025 \times 10^3 \text{ kg/m}^3)(9.81 \text{ m/s}^2)}$$
$$h = \frac{4.80 \times 10^6 \text{ Pa}}{(1.025 \times 10^3 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} = \boxed{477 \text{ m}}$$

5.  $h = 245 \text{ m}$   
 $P_0 = 1.01 \times 10^5 \text{ Pa}$   
 $\rho = 1.025 \times 10^3 \text{ kg/m}^3$   
 $g = 9.81 \text{ m/s}^2$

$$P = P_0 + \rho gh$$
$$P = 1.01 \times 10^5 \text{ Pa} + (1.025 \times 10^3 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(245 \text{ m})$$
$$P = 1.01 \times 10^5 \text{ Pa} + 2.46 \times 10^6 \text{ Pa}$$
$$P = \boxed{2.56 \times 10^6 \text{ Pa}}$$

6.  $h = 10\,916 \text{ m}$   
 $P_0 = 1.01 \times 10^5 \text{ Pa}$   
 $\rho = 1.025 \times 10^3 \text{ kg/m}^3$   
 $g = 9.81 \text{ m/s}^2$

$$P = P_0 + \rho gh = 1.01 \times 10^5 \text{ Pa} + (1.025 \times 10^3 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(10\,916 \text{ m})$$
$$P = 1.01 \times 10^5 \text{ Pa} + 1.10 \times 10^8 \text{ Pa} = \boxed{1.10 \times 10^8 \text{ Pa}}$$