NAME ______________________ DATE ______________ CLASS ______________

Fluid Mechanics

Problem B

PRESSURE

PROBLEM

The largest helicopter in the world, which was built in Russia, has a mass of 1.03×10^3 kg. If you placed this helicopter on a large piston of a hydraulic lift, what force would need to be applied to the small piston in order to slowly lift the helicopter? Assume that the weight of the helicopter is distributed evenly over the large piston's area, which is 1.40×10^2 m^2. The area of the small piston is 0.80 m^2.

SOLUTION

Given: $m = 1.03 \times 10^5$ kg

$A_1 = 1.40 \times 10^2$ m^2

$A_2 = 0.80$ m^2

$g = 9.81$ m/s^2

Unknown: $F_2 = ?$

Use the equation for pressure to equate the two opposing pressures in terms of force and area.

$$P = \frac{F}{A} \qquad\qquad P_1 = P_2$$

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$F_2 = F_1\frac{A_2}{A_1} = mg\left(\frac{A_2}{A_1}\right)$$

$$F_2 = (1.03 \times 10^5 \text{ kg})(9.81 \text{ m/s}^2)\left(\frac{0.80 \text{ m}^2}{1.40 \times 10^2 \text{ m}^2}\right)$$

$$F_2 = \boxed{5.8 \times 10^3 \text{ N}}$$

ADDITIONAL PRACTICE

1. Astronauts and cosmonauts have used pressurized spacesuits to explore the low-pressure regions of space. The pressure inside one of these suits must be close to that of Earth's atmosphere at sea level so that the space explorer may be safe and comfortable. The pressure on the outside of the suit is a fraction of 1.0 Pa. Clearly, pressurized suits are made of extremely sturdy material that can tolerate the stress from these pressure differences. If the average interior surface area of a pressurized spacesuit is 3.3 m^2, what is the force exerted on the suit's material? Assume that the pressure outside the suit is zero and that the pressure inside the suit is 1.01×10^5 Pa.

2. A strange idea to control volcanic eruptions is developed by a daydreaming engineer. The engineer imagines a giant piston that fits into the volcano's shaft, which leads from Earth's surface down to the magma chamber. The piston controls an eruption by exerting pressure that is equal to or greater than the pressure of the hot gases, ash, and magma that rise from the magma chamber through the shaft. The engineer assumes that the pressure of the volcanic material is 4.0×10^{11} Pa, which is the pressure in Earth's interior. If the material rises into a cylindrical shaft with a radius of 50.0 m, what force is needed on the other side of the piston to balance the pressure of the volcanic material?

3. The largest goat ever grown on a farm had a mass of 181 kg; on the other hand, the smallest "pygmy" goats have a mass of only about 16 kg. Imagine an agricultural show in which a large goat with a mass of 181 kg exerts a pressure on a hydraulic-lift piston that is equal to the pressure exerted by three pygmy goats, each of which has a mass of 16.0 kg. The area of the piston on which the large goat stands is 1.8 m^2. What is the area of the piston on which the pygmy goats stand?

4. The greatest load ever raised was the offshore Ekofisk complex in the North Sea. The complex, which had a mass of 4.0×10^7 kg, was raised 6.5 m by more than 100 hydraulic jacks. Imagine that his load could have been raised using a single huge hydraulic lift. If the load had been placed on the large piston and a force of 1.2×10^4 N had been applied to the small piston, which had an area of 5.0 m^2, what must the large piston's area have been?

5. The pressure that can exist in the interior of a star due to the weight of the outer layers of hot gas is typically several hundred billion times greater than the pressure exerted on Earth's surface by Earth's atmosphere. Suppose a pressure equal to that estimated for the sun's interior (2.0×10^{16} Pa) acts on a spherical surface within a star. If a force of 1.02 N $\times 10^{31}$ N produces this pressure, what is the area of the surface? What is the sphere's radius r? (Recall that a sphere's surface area equals $4\pi r^2$.)

6. The eye of a giant squid can be more than 35 cm in diameter—the largest eye of any animal. Giant squid also live at depths greater than a mile below the ocean's surface. At a depth of 2 km, the outer half of a giant squid's eye is acted on by an external force of 4.6×10^6 N. Assuming the squid's eye has a diameter of 38 cm, what is the pressure on the eye? (Hint: Treat the eye as a sphere.)

7. The largest tires in the world, which are used for certain huge dump trucks, have diameters of about 3.50 m. Suppose one of these tires has a volume of 5.25 m^3 and a surface area of 26.3 m^2. If a force of 1.58×10^7 N acts on the inner area of the tire, what is the absolute pressure inside the tire? What is the gauge pressure on the tire's surface?

Givens | *Solutions*

6. $h = 167$ m

$H = 1.50$ km

$\rho_{sw} = 1.025 \times 10^3$ kg/m^3

$$F_{g,i} = F_B$$

$$\rho_i V_i g = \rho_{sw} V_{sw} g$$

$$\rho_i (h + H) A g = \rho_{sw} H A g$$

$$\rho_i = \frac{\rho_{sw} H}{h + H}$$

$$\rho_i = \frac{(1.025 \times 10^3 \text{ kg/m}^3)(1.50 \times 10^3 \text{ m})}{167 \text{ m} + 1.50 \times 10^3 \text{ m}} = \frac{(1.025 \times 10^3 \text{ kg/m}^3)(1.50 \times 10^3 \text{ m})}{1670 \text{ m}} = \boxed{921 \text{ kg/m}^3}$$

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7. $\ell = 1.70 \times 10^2$ m

$r = \dfrac{13.9 \text{ m}}{2} = 6.95$ m

$m_{sw} = 2.65 \times 10^7$ kg

$a = 2.00$ m/s^2

$g = 9.81$ m/s^2

$$F_{net} = F_g - F_B$$

$$m_{sub} a = m_{sub} g - m_{sw} g$$

$$\rho_{sub} V a = \rho_{sub} V g - m_{sw} g$$

$$\rho_{sub} (g - a) V = m_{sw} g$$

$$\rho_{sub} = \frac{m_{sw} g}{(g - a) V} = \frac{m_{sw} g}{(g - a)(\pi r^2 \ell)}$$

$$\rho_{sub} = \frac{(2.65 \times 10^7 \text{ kg})(9.81 \text{ m/s}^2)}{(9.81 \text{ m/s}^2 - 2.00 \text{ m/s}^2)(\pi)(6.95 \text{ m})^2(1.70 \times 10^2 \text{ m})}$$

$$\rho_{sub} = \frac{(2.65 \times 10^7 \text{ kg})(9.81 \text{ m/s}^2)}{(9.81 \text{ m/s}^2)(\pi)(6.95 \text{ m})^2(1.70 \times 10^2 \text{ m})}$$

$$\rho_{sub} = \boxed{1.29 \times 10^3 \text{ kg/m}^3}$$

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8. $V = 6.00$ m^3

Δ apparent weight $= 800$ N

$\rho_{water} = 1.00 \times 10^3$ kg/m^3

$g = 9.81$ m/s^2

$$F_{g,1} = F_{g,2}$$

$F_{B,1}$ + apparent weight in water $= F_{B,2}$ + apparent weight in PEG solution

$\rho_{water} V g$ + apparent weight in water − apparent weight in PEG solution $= \rho_{soln} V g$

$$\rho_{soln} = \frac{\rho_{water} V g + \Delta \text{ apparent weight}}{V g}$$

$$\rho_{soln} = \frac{(1.00 \times 10^3 \text{ kg/m}^3)(6.00 \text{ m}^3)(9.81 \text{ m/s}^2) + 800 \text{ N}}{(6.00 \text{ m}^3)(9.81 \text{ m/s}^2)}$$

$$\rho_{soln} = \frac{5.89 \times 10^4 \text{ N} + 800 \text{ N}}{(6.00 \text{ m}^3)(9.81 \text{ m/s}^2)} = \frac{5.97 \times 10^4 \text{ N}}{(6.00 \text{ m}^3)(9.81 \text{ m/s}^2)}$$

$$\rho_{soln} = \boxed{1.01 \times 10^3 \text{ kg/m}^3}$$

Additional Practice B

1. $P = 1.01 \times 10^5$ Pa

$A = 3.3$ m^2

$$F = PA\ (1.01 \times 10^5 \text{ Pa})(3.3 \text{ m}^2) = \boxed{3.3 \times 10^5 \text{ N}}$$

Givens	*Solutions*
2. $P = 4.0 \times 10^{11}$ Pa $r = 50.0$ m	$F = PA = P(\pi r^2)$ $F = (4.0 \times 10^{11} \text{ Pa})(\pi)(50.0 \text{ m})^2$ $F = \boxed{3.1 \times 10^{15} \text{ N}}$
3. $m_1 = 181$ kg $m_2 = 16.0$ kg $A_1 = 1.8 \text{ m}^2$ $g = 9.81 \text{ m/s}^2$	$P_1 = P_2$ $\dfrac{F_1}{A_1} = \dfrac{F_2}{A_2}$ $A_2 = \dfrac{F_2 A_1}{F_1} = \dfrac{3m_2 g A_1}{m_1 g} = \dfrac{3m_2 A_1}{m_1}$ $A_2 = \dfrac{(3)(16.0 \text{ kg})(1.8 \text{ m}^2)}{181 \text{ kg}} = \boxed{0.48 \text{ m}^2}$
4. $m = 4.0 \times 10^7$ kg $F_2 = 1.2 \times 10^4$ N $A_2 = 5.0 \text{ m}^2$ $g = 9.81 \text{ m/s}^2$	$P_1 = P_2$ $\dfrac{F_1}{A_1} = \dfrac{F_2}{A_2}$ $A_1 = \dfrac{A_2 F_1}{F_2} = \dfrac{A_2 mg}{F_2}$ $A_1 = \dfrac{(5.0 \text{ m}^2)(4.0 \times 10^7 \text{ kg})(9.81 \text{ m/s}^2)}{1.2 \times 10^4 \text{ N}} = \boxed{1.6 \times 10^5 \text{ m}^2}$
5. $P = 2.0 \times 10^{16}$ Pa $F = 1.02 \times 10^{31}$ N	$A = \dfrac{F}{P} = \dfrac{1.02 \times 10^{31} \text{ N}}{2.0 \times 10^{16} \text{ Pa}} = \boxed{5.1 \times 10^{14} \text{ m}^2}$ $A = 4\pi r^2$ $r = \sqrt{\dfrac{A}{4\pi}} = \sqrt{\dfrac{5.1 \times 10^{14} \text{ m}^2}{4\pi}}$ $r = \boxed{6.4 \times 10^6 \text{ m}}$
6. $F = 4.6 \times 10^6$ N $r = \dfrac{38 \text{ cm}}{2} = 19$ cm	$P = \dfrac{F}{A}$ Assuming the squid's eye is a sphere, its total surface area is $4\pi r^2$. The outer half of the eye has an area of $A = 2\pi r^2$ $P = \dfrac{F}{2\pi r^2} = \dfrac{4.6 \times 10^6 \text{ N}}{(2\pi)(0.19 \text{ m})^2} = \boxed{2.0 \times 10^7 \text{ Pa}}$
7. $A = 26.3 \text{ m}^2$ $F = 1.58 \times 10^7$ N $P_o = 1.01 \times 10^5$ Pa	$P = \dfrac{F}{A} = \dfrac{1.58 \times 10^7 \text{ N}}{26.3 \text{ m}^2} = \boxed{6.01 \times 10^5 \text{ Pa}}$ $P_{gauge} = P - P_o = 6.01 \times 10^5 \text{ Pa} - 1.01 \times 10^5 \text{ Pa} = \boxed{5.00 \times 10^5 \text{ Pa}}$

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