NAME ______________________ DATE ____________ CLASS ______________

Fluid Mechanics

Problem A

BUOYANT FORCE

PROBLEM

The highest natural concentration of salts in water are found in the evaporating remnants of old oceans, such as the Dead Sea in Israel. Suppose a swimmer with a volume of 0.75 m^3 is able to float just beneath the surface of water with a density of 1.02×10^3 kg/m^3. How much extra mass can the swimmer carry and be able to float just beneath the surface of the Dead Sea, which has a density of 1.22×10^3 kg/m^3?

SOLUTION

1. DEFINE

Given: $V = 0.75 \text{ m}^3$

$\rho_1 = 1.02 \times 10^3 \text{ kg/m}^3$

$\rho_2 = 1.22 \times 10^3 \text{ kg/m}^3$

Unknown: $m' = ?$

2. PLAN

Choose the equation(s) or situation: In both bodies of water, the buoyant force equals the weight of the floating object.

$$F_{B,1} = F_{g,1}$$
$$\rho_1 V_g = m_g$$
$$F_{B,2} = F_{g,2}$$
$$\rho_2 V_g = (m + m')g = \rho_1 V_g + m'g$$

Rearrange the equation(s) to isolate the unknown(s):

$$m' = (\rho_2 - \rho_1)V$$

3. CALCULATE

Substitute the values into the equation(s) and solve:

$$m' = (1.22 \times 10^3 \text{ kg/m}^3 - 1.02 \times 10^3 \text{ kg/m}^3)(0.75 \text{ m}^3)$$
$$m' = (0.20 \times 10^3 \text{ kg/m}^3)(0.75 \text{ m}^3)$$
$$m' = \boxed{150 \text{ kg}}$$

4. EVALUATE

The mass that can be supported by buoyant force increases with the difference in fluid densities.

ADDITIONAL PRACTICE

1. The heaviest pig ever raised had a mass of 1158 kg. Suppose you placed this pig on a raft made of dry wood. The raft completely submerged in water so that the raft's top surface was just level with the surface of the lake. If the raft's volume was 3.40 m^3, what was the mass of the raft's dry wood? The density of fresh water is 1.00×10^3 kg/m^3.

2. *La Belle,* one of four ships that Robert La Salle used to establish a French colony late in the seventeenth century, sank off the coast of Texas. The ship's well-preserved remains were discovered and excavated in the 1990s. Among those remains was a small bronze cannon, called a minion. Suppose the minion's total volume is 4.14×10^{-2} m^3. What is the minion's mass if its apparent weight in sea water is 3.115×10^3 N? The density of sea water is 1.025×10^3 kg/m^3.

3. William Smith built a small submarine capable of diving as deep as 30.0 m. The submarine's volume can be approximated by that of a cylinder with a length of 3.00 m and a cross-sectional area of 0.500 m^2. Suppose this submarine dives in a freshwater river and then moves out to sea, which naturally consists of salt water. What mass of fresh water must be added to the ballast to keep the submarine submerged? The density of fresh water is 1.000×10^3 kg/m^3, and the density of sea water is 1.025×10^3 kg/m^3.

4. The largest iceberg ever observed had an area of 3.10×10^4 km^2, which is larger than the area of Belgium. If the top and bottom surfaces of the iceberg were flat and the thickness of the submerged part was 0.84 km, how large was the buoyant force acting on the iceberg? The density of sea water equals 1.025×10^3 kg/m^3.

5. A cannon built in 1868 in Russia could fire a cannonball with a mass of 4.80×10^2 kg and a radius of 0.250 m. When suspended from a scale and submerged in water, a cannonball of this type has an apparent weight of 4.07×10^3 N. How large is the buoyant force acting on the cannonball? The density of fresh water is 1.00×10^3 kg/m^3.

6. The tallest iceberg ever measured stood 167 m above the water. Suppose that both the top and the bottom of this iceberg were flat and the thickness of the submerged part was estimated to be 1.50 km. Calculate the density of the ice. The density of sea water equals 1.025×10^3 kg/m^3.

7. The Russian submarines of the "Typhoon" class are the largest submarines in the world. They have a length of 1.70×10^2 m and an average diameter of 13.9 m. When submerged, they displace 2.65×10^7 kg of sea water. Assume that these submarines have a simple cylindrical shape. If a "Typhoon"-class submarine has taken on enough ballast that it descends with a net acceleration of 2.00 m/s^2, what is the submarine's density during its descent?

8. To keep Robert La Salle's ship *La Belle* well preserved, shipbuilders are reconstructing the ship in a large tank filled with fresh water. Polyethylene glycol, or PEG, will then be slowly added to the water until a 30 percent PEG solution is formed. Suppose *La Belle* displaces 6.00 m^3 of liquid when submerged. If the ship's apparent weight decreases by 800 N as the PEG concentration increases from 0 to 30 percent, what is the density of the final PEG solution? The density of fresh water is 1.00×10^3 kg/m^3.

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Fluid Mechanics

Additional Practice A

Givens	Solutions
1. $m_p = 1158$ kg $V = 3.40$ m^3 $\rho = 1.00 \times 10^3$ kg/m^3 $g = 9.81$ m/s^2	$F_B = F_g$ $\rho V g = (m_p + m_r)g$ $m_r = \rho V - m_p = (1.00 \times 10^3 \text{ kg/m}^3)(3.40 \text{ m}^3) - 1158 \text{ kg} = 3.40 \times 10^3 \text{ kg} - 1158 \text{ kg}$ $m_r = \boxed{2.24 \times 10^3 \text{ kg}}$
2. $V = 4.14 \times 10^{-2}$ m^3 apparent weight = 3.115×10^3 N $\rho_{sw} = 1.025 \times 10^3$ kg/m^3 $g = 9.81$ m/s^2	$F_B = F_g$ − apparent weight $\rho_{sw} V g = mg$ − apparent weight $m = \rho_{sw} V + \dfrac{\text{apparent weight}}{g} = (1.025 \times 10^3 \text{ kg/m}^3)(4.14 \times 10^{-2} \text{ m}^3) + \dfrac{3.115 \times 10^3 \text{ N}}{9.81 \text{ m/s}^2}$ $m = 42.4 \text{ kg} + 318 \text{ kg} = \boxed{3.60 \times 10^2 \text{ kg}}$
3. $\ell = 3.00$ m $A = 0.500$ m^2 $\rho_{fw} = 1.000 \times 10^3$ kg/m^3 $\rho_{sw} = 1.025 \times 10^3$ kg/m^3	$F_{net,1} = F_{net,2} = 0$ $F_{B,1} - F_{g,1} = F_{B,2} - F_{g,2}$ $\rho_{fw} V g - mg = \rho_{sw} V g - (m + m_{ballast})g$ $m_{ballast} g = (\rho_{sw} - \rho_{fw}) V g$ $m_{ballast} = (\rho_{sw} - \rho_{fw}) A \ell$ $m_0 = (1.025 \times 10^3 \text{ kg/m}^3 - 1.000 \times 10^3 \text{ kg/m}^3)(0.500 \text{ m}^2)(3.00 \text{ m})$ $m_0 = (25 \text{ kg/m}^3)(0.500 \text{ m}^2)(3.00 \text{ m}) = \boxed{38 \text{ kg}}$
4. $A = 3.10 \times 10^4$ km^2 $h = 0.84$ km $\rho = 1.025 \times 10^3$ kg/m^3 $g = 9.81$ m/s^2	$F_B = \rho V g = \rho A h g$ $F_B = (1.025 \times 10^3 \text{ kg/m}^3)(3.10 \times 10^{10} \text{ m}^2)(840 \text{ m})(9.81 \text{ m/s}^2) = \boxed{2.6 \times 10^{17} \text{ N}}$
5. $m = 4.80 \times 10^2$ kg $g = 9.81$ m/s^2 apparent weight = 4.07×10^3 N	$F_B = F_g$ − apparent weight = mg − apparent weight $F_B = (4.80 \times 10^2 \text{ kg})(9.81 \text{ m/s}^2) - 4.07 \times 10^3 \text{ N} = 4.71 \times 10^3 \text{ N} - 4.07 \times 10^3 \text{ N}$ $F_B = \boxed{640 \text{ N}}$

II

Givens | *Solutions*

6. $h = 167$ m

$H = 1.50$ km

$\rho_{sw} = 1.025 \times 10^3$ kg/m^3

$$F_{g,i} = F_B$$

$$\rho_i V_i g = \rho_{sw} V_{sw} g$$

$$\rho_i (h + H)Ag = \rho_{sw} HAg$$

$$\rho_i = \frac{\rho_{sw} H}{h + H}$$

$$\rho_i = \frac{(1.025 \times 10^3 \text{ kg/m}^3)(1.50 \times 10^3 \text{ m})}{167 \text{ m} + 1.50 \times 10^3 \text{ m}} = \frac{(1.025 \times 10^3 \text{ kg/m}^3)(1.50 \times 10^3 \text{ m})}{1670 \text{ m}} = \boxed{921 \text{ kg/m}^3}$$

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7. $\ell = 1.70 \times 10^2$ m

$r = \dfrac{13.9 \text{ m}}{2} = 6.95$ m

$m_{sw} = 2.65 \times 10^7$ kg

$a = 2.00$ m/s^2

$g = 9.81$ m/s^2

$$F_{net} = F_g - F_B$$

$$m_{sub} a = m_{sub} g - m_{sw} g$$

$$\rho_{sub} Va = \rho_{sub} Vg - m_{sw} g$$

$$\rho_{sub}(g - a)V = m_{sw} g$$

$$\rho_{sub} = \frac{m_{sw} g}{(g - a)V} = \frac{m_{sw} g}{(g - a)(\pi r^2 \ell)}$$

$$\rho_{sub} = \frac{(2.65 \times 10^7 \text{ kg})(9.81 \text{ m/s}^2)}{(9.81 \text{ m/s}^2 - 2.00 \text{ m/s}^2)(\pi)(6.95 \text{ m})^2(1.70 \times 10^2 \text{ m})}$$

$$\rho_{sub} = \frac{(2.65 \times 10^7 \text{ kg})(9.81 \text{ m/s}^2)}{(9.81 \text{ m/s}^2)(\pi)(6.95 \text{ m})^2(1.70 \times 10^2 \text{ m})}$$

$$\rho_{sub} = \boxed{1.29 \times 10^3 \text{ kg/m}^3}$$

II

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8. $V = 6.00$ m^3

Δ apparent weight $= 800$ N

$\rho_{water} = 1.00 \times 10^3$ kg/m^3

$g = 9.81$ m/s^2

$$F_{g,1} = F_{g,2}$$

$F_{B,1}$ + apparent weight in water = $F_{B,2}$ + apparent weight in PEG solution

$\rho_{water} Vg$ + apparent weight in water − apparent weight in PEG solution = $\rho_{soln} Vg$

$$\rho_{soln} = \frac{\rho_{water} Vg + \Delta \text{ apparent weight}}{Vg}$$

$$\rho_{soln} = \frac{(1.00 \times 10^3 \text{ kg/m}^3)(6.00 \text{ m}^3)(9.81 \text{ m/s}^2) + 800 \text{ N}}{(6.00 \text{ m}^3)(9.81 \text{ m/s}^2)}$$

$$\rho_{soln} = \frac{5.89 \times 10^4 \text{ N} + 800 \text{ N}}{(6.00 \text{ m}^3)(9.81 \text{ m/s}^2)} = \frac{5.97 \times 10^4 \text{ N}}{(6.00 \text{ m}^3)(9.81 \text{ m/s}^2)}$$

$$\rho_{soln} = \boxed{1.01 \times 10^3 \text{ kg/m}^3}$$

Additional Practice B

1. $P = 1.01 \times 10^5$ Pa

$A = 3.3$ m^2

$$F = PA\ (1.01 \times 10^5 \text{ Pa})(3.3 \text{ m}^2) = \boxed{3.3 \times 10^5 \text{ N}}$$